



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIORSERTIFIKAAT***

GRADE/GRAAD 12

JUNE/JUNIE 2023

**MATHEMATICS P2/WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 12 pages.
Hierdie nasienriglyn bestaan uit 12 bladsye.

$$2.5. \bar{x} - M = 16,8 - 16 \\ = 0,8 \quad \checkmark$$

\bar{x} and M lie in the modal interval / class. \checkmark

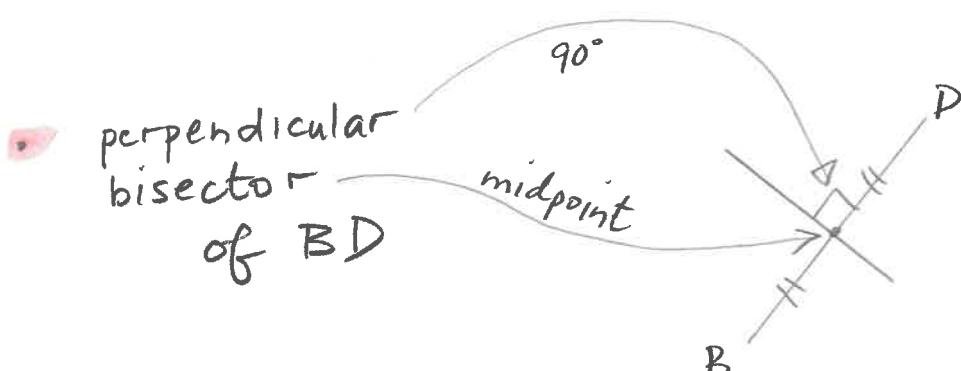
\checkmark data is almost symmetrical, just slightly positively (to the right) skewed.

3

QUESTION 3/VRAAG 3

3.1	$y = -\frac{1}{3}x + 10 \quad \checkmark \quad (m_1 \times m_2 = -1)$ $\frac{p}{2} = -\frac{1}{3}(4) + 10 \quad \checkmark$ $p = \frac{52}{3}$ $\leftarrow \rightarrow \quad 17,33$	<ul style="list-style-type: none"> ✓ equation of line / verg. van lyn ✓ substitution of point / vervanging van punt ✓ answer / antwoord 	(3)
3.2	$O(0;0)$ and en $P(-2; p-1)$ and en $OP = \sqrt{(-2-0)^2 + (p-1-0)^2} \quad \checkmark$ $OP^2 = (-2-0)^2 + (p-1-0)^2$ $(2p)^2 = 4 + p^2 - 2p + 1 \quad \checkmark$ $4p^2 = p^2 - 2p + 5$ $3p^2 + 2p - 5 = 0 \quad \checkmark$ $(3p+5)(p-1) = 0 \quad \checkmark$ $p = \cancel{\frac{5}{3}} \text{ reject or } p = 1$ $\therefore p = 1 \quad \checkmark$	<ul style="list-style-type: none"> ✓ substitution / vervanging ✓ simplification / vereenvoudiging ✓ standard form / standaardvorm ✓ factors / faktore ✓ answer / antwoord with selection 	(5)
3.3.1	$m_{BD} = 2 \quad \checkmark \quad \frac{-5-3}{-2-2} \quad B(2,3) \quad D(-2,-5) \quad 2$	✓✓ answer / antwoord	(2)
3.3.2	Midpoint of BD : Middelpunt van BD $(0; -1)$ $y = -\frac{1}{2}x - 1 \quad \checkmark$ $x_M = \frac{2+(-2)}{2} = 0 \quad \checkmark$ $y_M = \frac{3+(-5)}{2} = -1 \quad \checkmark$	<ul style="list-style-type: none"> ✓✓ midpoint / middelpunt ✓ answer / antwoord 	(3)
3.3.3	$x^2 + y^2 = 25 \quad \checkmark \quad \checkmark$	✓✓ answer / antwoord	(2)
3.3.4	$x^2 + \left(-\frac{1}{2}x - 1\right)^2 = 25 \quad \checkmark$ $x^2 + \frac{1}{4}x^2 + x + 1 = 25 \quad \checkmark$ $4x^2 + x^2 + 4x + 4 = 100$ $5x^2 + 4x - 96 = 0 \quad \checkmark$ $(5x+24)(x-4) = 0 \quad \checkmark$ $x = -\frac{24}{5} \quad \text{or/of} \quad x = 4 \quad \checkmark$ $\therefore y = \frac{7}{5} \quad \checkmark$	<ul style="list-style-type: none"> ✓ substitution / vervanging ✓ simplification / vereenvoudiging ✓ standard form / standaardvorm ✓ factorisation / faktorisering ✓ x-values / x-waardes ✓ answer / antwoord 	(6)
			[21]

$$-\frac{1}{2} \left(-\frac{24}{5}\right) - 1$$



3.1.

OR

$$m_1 = 3$$

$$m_2 = -\frac{1}{3} \quad \perp$$

$$\checkmark \frac{10 - P/2}{0 - 4} = -\frac{1}{3} \checkmark$$

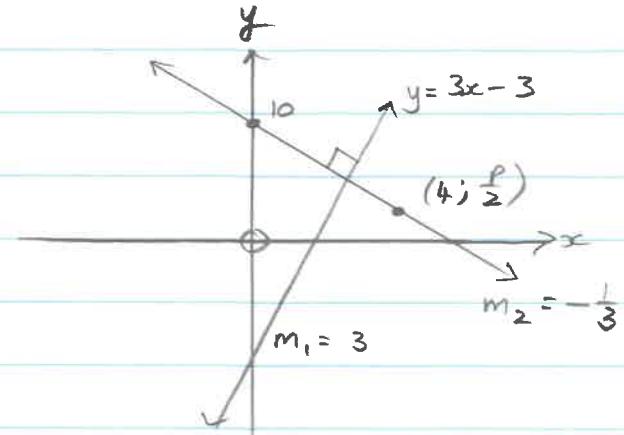
$$10 - P/2 = \frac{4}{3}$$

$$-P/2 = -\frac{26}{3}$$

$$\underline{P = \frac{52}{3}} \checkmark$$

$$17, 33$$

3



$$3.3.4. \quad y = -\frac{1}{2}x - 1$$

$$y + 1 = -\frac{1}{2}x$$

$$-2y - 2 = x \checkmark$$

$$x^2 + y^2 = 25$$

$$(-2y - 2)^2 + y^2 = 25 \checkmark$$

$$4y^2 + 8y + 4 + y^2 = 25 \checkmark$$

$$5y^2 + 8y - 21 = 0 \checkmark$$

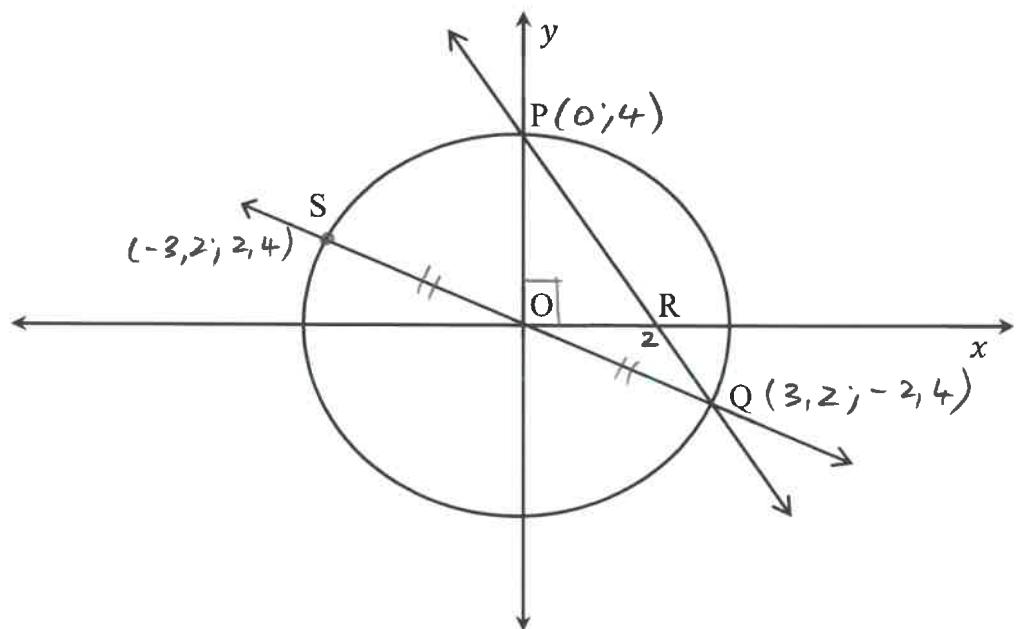
$$(y + 3)(5y - 7) = 0 \checkmark$$

$$\therefore y = -3 \text{ or } \frac{7}{5}$$

$$\therefore y_C = \frac{7}{5} \checkmark \quad 6$$

3.1.

QUESTION 4 / VRAAG 4



4.1	$x^2 + y^2 = 16$ $x^2 + (4 - 2x)^2 = 16$ $x^2 + 16 - 16x + 4x^2 = 16$ $5x^2 - 16x = 0$ $x(5x - 16) = 0$ $x = 0 \quad \text{or/of} \quad x = \frac{16}{5} = 3,2$ $y = 4 - 2(0) \quad y = 4 - 2(3,2)$ $y = 4 \quad y = -2,4$	✓ $y = 4 - 2x$ ✓ substitution / vervanging ✓ standard form / standaardvorm ✓ factors / faktore ✓ x -values / x -waardes ✓ substitution / vervanging ✓ y -values / y -waardes	7
4.2	$S(-3, 2; 2, 4)$ $\frac{x_1 + x_2}{2} = 0 \quad \frac{y_1 + y_2}{2} = 0$ radius	✓✓ answer / antwoord	2 (2)
4.3	$y = 4 - 2x$ $0 = 4 - 2x$ $2x = 4$ $x = 2$ $R(2, 0)$ $\therefore \text{radius} = 2 \text{ units} / \text{eenhede}$ $(x - 2)^2 + y^2 = 4$	✓ equating to 0 / gelyk stel aan 0 ✓ $x = 2$ ✓ radius = 2 units/eenhede ✓ answer / antwoord	4 (4)
4.4	$(x - y)^2 + y^2 - y = 12$ $(x - 6)^2 + y^2 - y + \frac{1}{4} = 12 + \frac{1}{4}$ $(x - 6)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{49}{4}$ Centre/Middelpunt $\left(6; \frac{1}{2}\right)$ and/en $O(0; 0)$ $d = \sqrt{(6-0)^2 + \left(\frac{1}{2}-0\right)^2} = \sqrt{\frac{145}{4}}$ $d \approx 6,02$	✓ completing the square / vierkantsvoltooiing ✓ simplification / vereenvoudiging ✓ coordinates of midpoint / koördinate van middelpunt ✓ substitution in distance formula / vervanging in afstand formule ✓ answer / antwoord	5 (5)
			[18]

QUESTION 5/VRAAG 5

NB correct labelling of θ and α

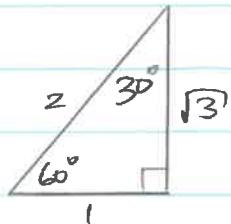
5.1	$5 \cos \theta - 3 = 0$ $\cos \theta = \frac{3}{5}$ ✓ θ Q IV $(3, -4)$ ✓	$17 \sin \alpha = 8$ $\sin \alpha = \frac{8}{17}$ ✓ α C II	<ul style="list-style-type: none"> ✓ $\cos \theta = \frac{3}{5}$ measured from direction of +ve x-axis ✓ $\sin \alpha = \frac{8}{17}$ anti clockwise ✓ -4 in correct quadrant in korrekte kwadrant ✓ -15 in correct quadrant in korrekte kwadrant ✓ correct values / korrekte waardes ✓ answer / antwoord
	$\begin{aligned} \tan \alpha + \tan \theta \\ = -\frac{8}{15} + \left(-\frac{4}{3}\right) \end{aligned}$ $\begin{aligned} &= -\frac{28}{15} \\ &= -\frac{14}{15} \end{aligned}$ ✓ both $\frac{8}{-15}$ $-\frac{4}{3}$	6	(6)
5.2	$\cos 42^\circ = p$		
5.2.1	$\sin 48^\circ = p$ ✓ $\sin(180^\circ - 42^\circ) = \cos 42^\circ$	✓✓ answer / antwoord	2
5.2.2	$\sin(-2202^\circ)$ 0.2 $= \sin(-42^\circ)$ $= -\sin 42^\circ$ ✓ $= -\sqrt{1-p^2}$ ✓	$\sin 318^\circ$ $= \sin(360^\circ - 42^\circ)$ $= -\sin 42^\circ$ $= \frac{\sqrt{1-p^2}}{1}$	✓ -sin(42°) ✓ answer / antwoord
5.2.3	$\cos 84^\circ$ $= \cos 2(42^\circ)$ ✓ $= 2 \cos^2 42^\circ - 1$ $= 2p^2 - 1$ ✓	$\cos 2x = 2\cos^2 x - 1$ $x = 42^\circ$ $\cos 84^\circ = 2\cos^2 42^\circ - 1$	✓ cos 2(42°) ✓ answer / antwoord
5.3	$\frac{\tan 300^\circ + \cos(90^\circ + x)}{\sin(180^\circ - x) + 2 \cos(-30^\circ)}$ $= \frac{-\tan 60^\circ - \sin x}{\sin x + 2 \left(\frac{\sqrt{3}}{2}\right)}$ $= \frac{-\sqrt{3} - \sin x}{\sin x + \sqrt{3}}$ $= \frac{-(\sin x + \sqrt{3})}{(\sin x + \sqrt{3})}$ $= -1$		✓ -tan 60° ✓ -sin x ✓ sin x ✓ $\frac{\sqrt{3}}{2}$ ✓ taking out of negative sign. uithaal van negatiewe teken ✓ answer / antwoord

5.3.

$$\frac{\tan 30^\circ + \cos(90^\circ + x)}{\sin(180^\circ - x) + 2\cos(-30^\circ)}$$

$$\cdot \tan 30^\circ = \tan(30^\circ - 60^\circ) = -\tan 60^\circ \quad \checkmark$$

$$\begin{aligned} &= -\frac{a}{a} \\ &= -\frac{\sqrt{3}}{1} \\ &= -\sqrt{3} \end{aligned}$$



$$\cdot \cos(90^\circ + x) = -\sin x$$

$$\cdot \sin(180^\circ - x) = \sin x$$

$$\cdot \cos(-30^\circ) = \cos 30^\circ$$

$$\begin{aligned} &= \frac{a}{h} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\therefore \frac{-\sqrt{3} + (-\sin x)}{\sin x + 2(\frac{\sqrt{3}}{2})} \quad \checkmark$$

$$= \frac{-\sqrt{3} - \sin x}{\sin x + \sqrt{3}}$$

$$= \frac{-(\sqrt{3} + \sin x)}{\sin x + \sqrt{3}} \quad \checkmark$$

$$= \frac{-1}{\downarrow} \quad \checkmark$$

6

5.4	$\frac{1 - \sin 2x}{\cos 2x} = \frac{\cos x - \sin x}{\cos x + \sin x}$ $LHS = \frac{1 - 2 \sin x \cos x}{\cos^2 x - \sin^2 x}$ $LHS = \frac{\cos^2 x - 2 \sin x \cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$ $LHS = \frac{(\cos x - \sin x)(\cos x - \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $LHS = \frac{\cos x - \sin x}{\cos x + \sin x}$	✓ $2 \sin x \cos x$ ✓ $\cos^2 x - \sin^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$ ✓ factorising / faktorisering ✓ factorising / faktorisering	5 (5)
5.5	$\cos x - \sin x = \sqrt{2}$ $\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = 1$ $\cos 45^\circ \cos x - \sin 45^\circ \sin x = 1$ $\cos(45^\circ + x) = 1$ $45^\circ + x = 360^\circ \cdot k$ $x = -45^\circ + 360^\circ \cdot k ; k \in \mathbb{Z}$	✓ division by / deling deur $\sqrt{2}$ ✓ $\frac{1}{\sqrt{2}} = \cos 45^\circ / \sin 45^\circ$ ✓ expansion rule / reël ✓ $45^\circ + x = 360^\circ \cdot k$ ✓ answer / antwoord	(5)
			[28]

55.

(OR)

$$\cos x - \sin x = \sqrt{2}$$

$$(\cos x - \sin x)^2 = (\sqrt{2})^2$$

$$(\)^2 \text{ b.s. } \therefore \text{check solns}$$

$$\cos^2 x - 2 \sin x \cos x + \sin^2 x = 2 \quad \checkmark$$

$$\checkmark 1 - 2 \sin x \cos x = 2$$

$$-\sin 2x = 1$$

$$\sin 2x = -1 \quad \checkmark$$

$$\sin A = -1$$

$$A = 270^\circ + k \cdot 360^\circ$$

$$2x = 270^\circ + k \cdot 360^\circ$$

$$x = 135^\circ + k \cdot 180^\circ \quad \checkmark$$

$$A = 2x$$

$$\dots; -225^\circ; -45^\circ; 135^\circ; 315^\circ; 45^\circ; 675^\circ; \dots \text{ check}$$

$$\therefore \dots; -45^\circ; 315^\circ; 675^\circ; \dots$$

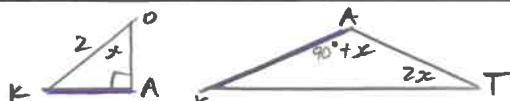
So $x = 315^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ ✓

5

QUESTION 6/VRAAG 6

6.1		<ul style="list-style-type: none"> ✓ shape / vorm ✓ intercepts / afsnitte ✓ starting and end points / begin en eindpunte 	
6.2	Period / Periode = 180°	1	✓ answer / antwoord (1)
6.3	$h(x) = \tan x + 2$	1	✓ answer / antwoord (1)
6.4	$-135^\circ \leq x < -90^\circ$	3	$\cos 2x \leq \tan x - 1$ $y_g \leq y_f$ ✓ answer / antwoord (3)
6.5	$\cos B + 1 = \tan \frac{1}{2}B$ Let/Laat $B = 2x$ $\cos 2x + 1 = \tan \frac{1}{2}(2x)$ $\cos 2x = \tan x - 1$ $x = -135^\circ$ and/en $x = 45^\circ$ $\therefore B = -270^\circ$ and/en $B = 90^\circ$	4	$\checkmark \cos 2x + 1 = \tan \frac{1}{2}(2x)$ $\checkmark \cos 2x = \tan x - 1$ \checkmark both x -values <i>beide x-waardes</i> \checkmark both B values / <i>beide B waardes</i> (4)
			[12]

QUESTION 7/VRAAG 7

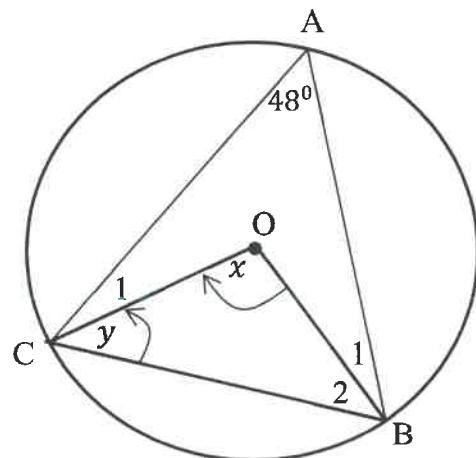


7.1	$\text{In } \triangle OAK$ $\sin x = \frac{AK}{2}$ ✓ $AK = 2 \sin x$ ✓ $2 \sin x = 2 KT \sin x$ $KT = 1$ \Rightarrow	$\text{In } \triangle KAT$ $\frac{AK}{\sin 2x} = \frac{KT}{\sin(90^\circ + x)}$ $AK = \frac{KT 2 \sin x \cos x}{\cos x}$ $AK = 2KT \sin x$	✓ $\sin x = \frac{AK}{2}$ ✓ $AK = 2 \sin x$ ✓ use of sine rule / gebruik van sinusreël ✓ $AK = 2KT \sin x$ 5 ✓ $KT = 1$	(5)
	$\text{In } \triangle KAT$ $T\hat{K}A = 90^\circ - 3x$ ✓ $AT = \frac{1}{\sin(90^\circ - 3x)} = \frac{1}{\sin(90^\circ + x)}$ $AT = \frac{\cos 3x}{\cos x}$	$T\hat{K}A = 90^\circ - 3x$ ✓ use of sine rule gebruik van sinusreël 2		
7.3	$AT = \frac{\cos 3x}{\cos x}$ $AT = \frac{\cos(2x + x)}{\cos x}$ $AT = \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$ ✓ $AT = \frac{\cos 2x \cos x - 2 \sin x \cos x \sin x}{\cos x}$ $AT = \frac{\cos x(\cos 2x - 2 \sin^2 x)}{\cos x}$ ✓ $AT = \frac{\cos x}{1 - 2 \sin^2 x} - 2 \sin^2 x$ ✓ $AT = \frac{\cos x}{1 - 4 \sin^2 x}$ \Rightarrow ✓	✓ splitting of $\cos 3x$ and expansion opbreek van $\cos 3x$ en uitbreiding ✓ common factor / gemene faktor ✓ expansion of $\cos 2x$ uitbreiding van $\cos 2x$ ✓ answer / antwoord		(4)
				[11]

• $90^\circ + x + 2x + T\hat{K}A = 180^\circ$ sum \therefore in $\triangle = 180^\circ$
 $T\hat{K}A = 90^\circ - 3x$

QUESTION 8/VRAAG 8

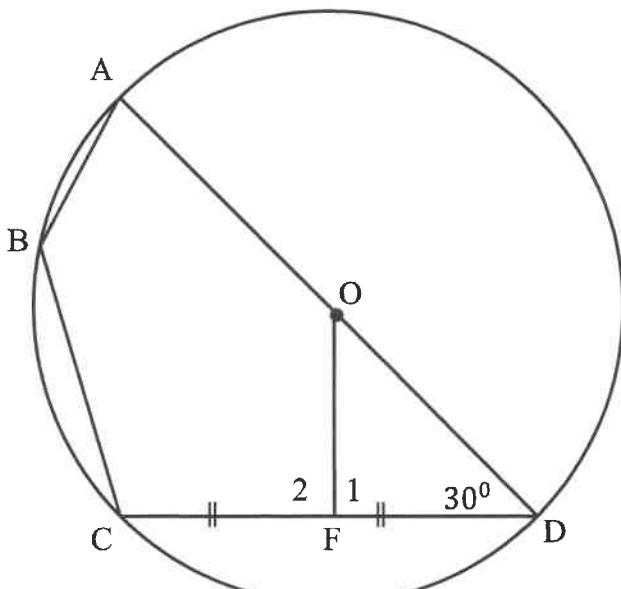
8.1


✓S ✓R

8.1.1 $x = 96^\circ$ [∠ at centre = $2 \times$ ∠ at circumference]
→ [middelpunte ∠ = $2 \times$ omtrekshoek] 2 (2)

8.1.2 $\hat{B}_2 = y$ ✓SR [angles opp = sides], radii
[hoeke teenoor = sye]
 $2y + 96^\circ = 180^\circ$ [sum of angles of Δ]
[som van die hoeke van Δ] 2 (2)
 $y = 42^\circ$ ✓SP

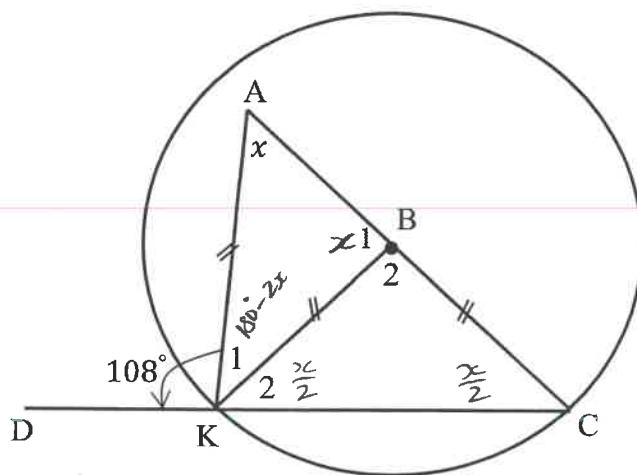
8.2


✓S ✓R

8.2.1 $\hat{F}_1 = 90^\circ$ [line from centre O to midpt of chord]
→ [lyn vanaf middelpunt halveer koord] ✓ S ✓ R 2 (2)

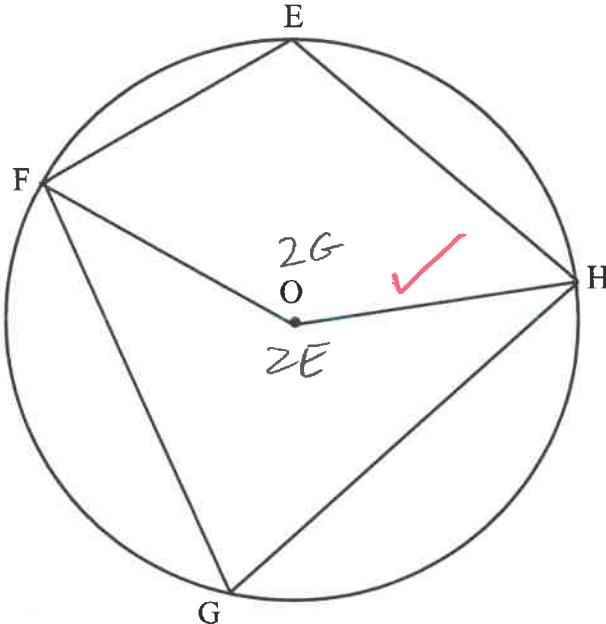
8.2.2 $A\hat{B}C = 150^\circ$ [opp angles of cyclic quad]
→ [teenoorst. hoeke van koordevierhoek] ✓ S ✓ R 2 (2)
✓R

8.3



8.3.1	$\hat{B}_1 = x$ ✓ ✓ [angles opp equal sides] [hoeke teenoor gelyke sye]	✓S ✓R 2	(2)
8.3.2	$\hat{K}_2 + \hat{C} = x$ ✓ ✓ [ext angle of Δ] / [buitehoek van Δ] $\hat{C} = \hat{K}_2$ ✓ S/R [angles opp equal sides] [hoeke teenoor gelyke sye] $\hat{C} = \frac{x}{2}$	✓S ✓R ✓S/R 3	
8.3.3	$\hat{K}_1 = 180^\circ - 2x$ [sum of angles of Δ] ✓ S/R $108^\circ + 180^\circ - 2x + \frac{x}{2} = 180^\circ$ ✓ [angles on str line] [hoeke op reguitlyn] $x = 72^\circ$	✓S/R ✓S ✓R ✓ answer / antwoord 4	(4) [17]

QUESTION 9 / VRAAG 9



Construct radii OF and OH.

$$F\hat{O}H = 2 \times \widehat{G}$$

✓SF [angle at centre = $2 \times$ angle at circum]

$$\text{Reflex } F\hat{O}H = 2 \times \widehat{E}$$

✓SF [angle at centre = $2 \times$ angle at circum]

$$F\hat{O}H + F\hat{O}H = 2\widehat{G} + 2\widehat{E}$$

$$\text{but } F\hat{O}H + F\hat{O}H = 360^\circ$$

$$2\widehat{G} + 2\widehat{E} = 360^\circ$$

$$\widehat{G} + \widehat{E} = 180^\circ$$

Trek radiusse OF en OH.

$$F\hat{O}H = 2 \times \widehat{G}$$

[middelpunte \angle = $2 \times$ omtrekshoek]

$$\text{Omwenteling } F\hat{O}H = 2 \times \widehat{E}$$

[middelpunte \angle = $2 \times$ omtrekshoek]

$$F\hat{O}H + F\hat{O}H = 2\widehat{G} + 2\widehat{E}$$

$$\text{maar } F\hat{O}H + F\hat{O}H = 360^\circ$$

[hoeke rondom 'n punt]

$$2\widehat{G} + 2\widehat{E} = 360^\circ$$

$$\widehat{G} + \widehat{E} = 180^\circ$$

✓S

✓S/R

✓S/R

✓S

✓S/R

✓S

6

✓S

✓S/R

✓S/R

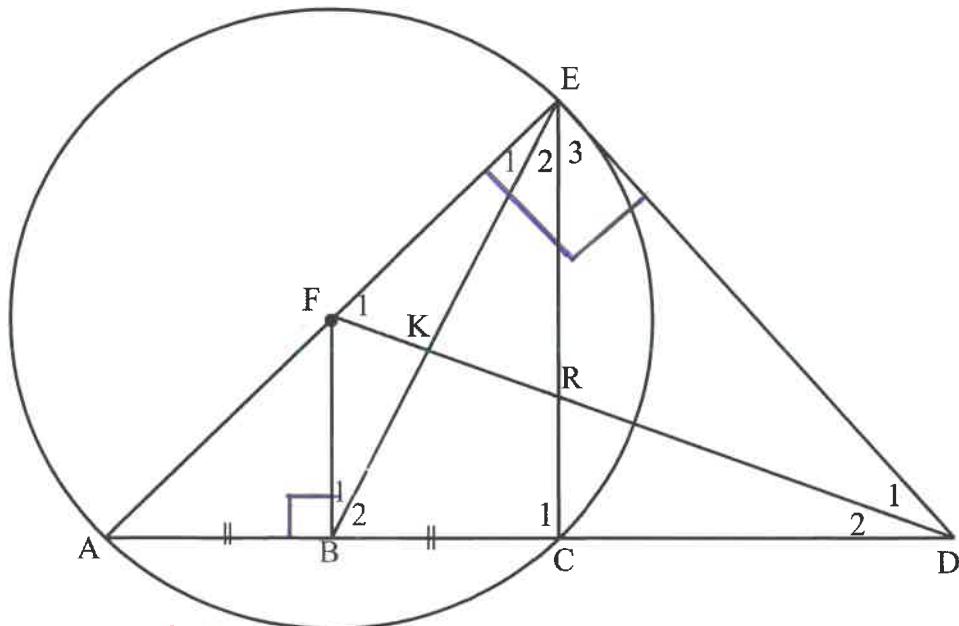
✓S

✓S/R

✓S

[6]

QUESTION 10/VRAAG 10



10.1	$FED = 90^\circ$ ✓ [radius \perp tan] / [radius \perp raaklyn] $FBA = 90^\circ$ ✓ [line from centre O to midpt of chord] $\therefore FED = FBA$ both $= 90^\circ$ $\therefore EFBD = \text{cyclic quad}$ <u>congr ext</u> \wedge cyclic quad $\hat{F} = \hat{B}$ ✓ R	✓ S ✓ R ✓ S/R ✓ R	4
10.2	$\hat{C}_1 = 90^\circ$ ✓ [angle in semi-circle] / [hoek in semi-sirkel] In $\Delta s B_2 C_1 E_2, F_1, E_1 + 2 + 3 D_1$, $\hat{C}_1 = FED [90^\circ]$ $\hat{B}_2 = \hat{F}_1$ ✓ [angles in same segment] $[hoeke in dieselfde segment]$ $\Delta BCE \parallel\!\! \Delta FED$ <u>$\hat{C}_1 = \hat{F}_1$</u> ✓ R	✓ S/R ✓ S/R ✓ S ✓ R ✓ S ✓ R	6
10.3	$\frac{BC}{FE} = \frac{CE}{ED}$ ✓ ✓ R $\Delta BCE \parallel\!\! \Delta FED$ $FE = FA$ ✓ SR radii $\therefore BC = \frac{FA \cdot CE}{ED}$	✓ S ✓ R ✓ SR	3
10.4	$FBA = \hat{C}_1$ [both $= 90^\circ$] $\therefore FB \parallel CE$ [corresponding angles equal] $[ooreenkomsige hoeke gelyk]$ $\frac{AC}{BC} = \frac{AE}{FE}$ [line parallel to one side of triangle] $[lyn ewewydig aan een sy van driehoek]$ $BC = \frac{AC \cdot FE}{AE}$	✓ S/R ✓ S/R ✓ S ✓ R	4
			[17]
		TOTAL/TOTAAL:	150

$$10.4. \quad AF = FC \quad \text{radius}$$

$$AB = BC \quad \text{given}$$

$\therefore BF \parallel CE$ ✓^S ✓^R midpt thm

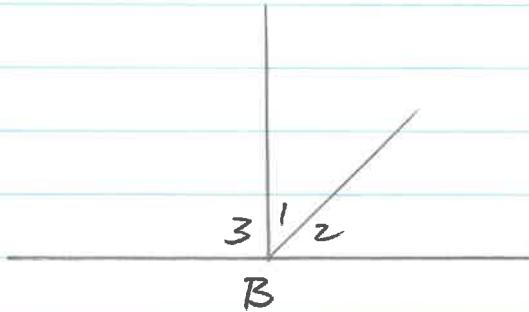
$$\therefore \frac{BC}{AC} = \frac{FE}{AE} \quad \checkmark \checkmark \text{ line } \parallel \text{ to 1 side of } \triangle$$

$$BC = \frac{AC \cdot FE}{AE}$$

→ D

4

NB SUBDIVIDED ANGLES



Please be specific

$\hat{B}_1, \hat{B}_2, \hat{B}_3, \hat{B}_1 + \hat{B}_2$ etc

NOT JUST \hat{B} !!!

I mark what I see : not what you may be thinking !!!